

WIND TURBULENCE RESPONSE OF MOORED AEROSTATS

BY MODAL ANALYSIS*

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Abstract

In order to predict nose loads and mooring line loads on moored aerostats, a dynamic FEA (finite element analysis) model was developed. The analytical model includes the mass and elastic properties of the mooring system, the aerostat, and the interface (nose structure and mooring lines). The damping included both aerodynamic damping of the aerostat and viscous damping of the mooring system. The modal approach was used where modal vectors were computed in the frequency region of interest. The transmissibility of the system was then determined for mooring lines and nose reactions. Wind turbulence was applied as a random spectrum giving spectra of displacements and forces. This analysis provides valuable insight into the performance of a moored aerostat in turbulent winds and design loads for the nose structure and mooring lines.

Introduction

In addition to buoyant lift and aerodynamic lift and drag due to steady winds, moored aerostats and airships are

subject to wind gusts and turbulence. Design to these conditions requires system dynamic analysis. The purpose of this study is to analyze the stresses and deflections developed in a moored aerostat or airship subjected to turbulent wind loading

Two basically different approaches for evaluating structural response are available: time domain and frequency domain analysis. Time domain or dynamic simulation of moored aerostats has been addressed in previous papers¹ in which the aerostat was subjected to a prescribed time dependent loading. Modal analysis or stability analysis of tethered aerostats in flight goes back to the work of Tracy Redd², James DeLaurier³, and Sam Jones⁴.

This paper presents the frequency domain or modal method of analysis performed with Finite Element Analysis (FEA). The time variation of the wind is generally not completely known but can be defined statistically as a random dynamic loading. The modal method provides statistical information about the stresses and deflections. This method also gives additional insight into the dynamic behavior of the system.

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Theoretical Background

Modal Method of Dynamic Analysis

The equation of motion for a linear dynamic system is as defined in Equation (1) where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, $\{f(t)\}$ is a time varying load vector, and $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are the acceleration, velocity, and displacement vectors, respectively.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{f(t)\} \quad (1)$$

Equation (1) can be solved by time domain or frequency domain methods. For the frequency domain, start by considering undamped free vibration by setting the $[C]$ and $\{f\}$ matrices to zero. Assume that all parts of the structure are vibrating harmonically at the same frequency, then the displacement vector is Equation (2).

$$\{u\} = \{f\}e^{i\omega t} \quad (2)$$

Substituting (2) into Equation (1) leads to the basic equation of frequency analysis, Equation (3). This equation is satisfied at a discrete set of ω 's called *modal* or *natural* frequencies. The squares of the natural frequencies are also called eigenvalues. Corresponding to each ω_i there is a vector, $\{f_i\}$, called a modal vector, mode shape, or eigenvector. These vibration modes of a structure provide insight for identifying its mechanical behavior.

$$[[K] - \omega^2[M]]\{f\} = 0 \quad (3)$$

The lower frequency modes are especially important for the moored aerostat dynamic system. Areas in the structure with large amplitudes in the lower modes will usually have the largest displacements when the structure is loaded either dynamically or statically. The higher frequency modes can generally be ignored because they have a negligible effect on the overall system performance.

The mode shapes are used to relate the physical coordinates of the system to *modal* or *generalized* coordinates. The mass, damping, stiffness, and load matrices can also be transformed to modal coordinates. If the matrices are orthogonal, then the system of equations can be decoupled into a series of single degree of freedom equations, one for each retained mode. The total response of the system is then found by superimposing

the response of each individual mode. This is the basis for the uncoupled modal method of dynamic response analysis.

Frequency and Random Response

If the excitation, x , of the system is steady state simple harmonic motion, then the response, y , is also steady state simple harmonic motion at the same frequency, Equation (4).

$$y = H(\omega)x = H(\omega)e^{i\omega t} \quad (4)$$

$H(\omega)$ is called the *frequency response function* or *transfer function* which relates the amplitude and phase of the response with the excitation. The frequency response function can be found by substituting Equation (4) into the equation of motion, (1), and solving for $H(\omega)$. The response can be any quantity (displacement, velocity, stress, etc.) at any point in a complex structure, and the excitation can be any quantity (force, acceleration, etc.) at any other point on the structure.

Further, if the excitation is a spectral density function, $S_x(\omega)$, the response will also be a spectral density function, $S_y(\omega)$. These are related as shown in Equation (5).

$$S_y(\omega) = |H(\omega)|^2 S_x(\omega) \quad (5)$$

The mean squared value of the response can be found by integrating the spectral density function over all frequencies. The root mean squared (RMS) value is then given by Equation (6).

$$RMS = \sqrt{\int_{-\infty}^{\infty} S(\omega) d\omega} \quad (6)$$

Wind Spectral Density Function

The wind turbulence used for this analysis is statistically described by the Dryden velocity spectral density function as described in References ^{5,6}. The spectrum, $S(\omega)$, is defined in Equation (7). The units are velocity squared per radian frequency band, i.e. $(m/s)^2$ per (radian per second). The subscripts u , v , and w represent the longitudinal, lateral, and vertical directions respectively. L is the scale of turbulence, u_0 is the mean wind speed, ω is the angular frequency, and s^2 is the variance of the turbulence.

$$\begin{Bmatrix} S_u(\mathbf{w}) \\ S_v(\mathbf{w}) \\ S_w(\mathbf{w}) \end{Bmatrix} = \frac{2}{u_0 P} \begin{Bmatrix} L_u S_u^2 \frac{1}{1 + (L_u \mathbf{w} / u_0)^2} \\ L_v S_v^2 \frac{1 + 3(L_v \mathbf{w} / u_0)^2}{[1 + (L_v \mathbf{w} / u_0)^2]^2} \\ L_w S_w^2 \frac{1 + 3(L_w \mathbf{w} / u_0)^2}{[1 + (L_w \mathbf{w} / u_0)^2]^2} \end{Bmatrix} \quad (7)$$

The scale of turbulence is a function of height above the surface. For heights of up to 18.3 meters (the heights of moored aerostat), the values of L 's have values of 170, 98, and 53 meters for the u , v , and w directions. If the value of s is specified in the u direction, then the v and w turbulence is reduced:

$$\mathbf{S}_v = (1.70/2.33)\mathbf{S}_u \quad \text{and} \quad \mathbf{S}_w = (1.25/2.33)\mathbf{S}_u$$

for the 0 to 18.3-meter height.

The forces due to wind are given by Equation (8). X , Y , and Z represent the forces along the u , v , and w directions, respectively. S_{ref} is the reference area, u_0 is the free-stream velocity, ρ is the mass density of the air, u , v , and w are the wind variations along the u , v , and w directions; and \hat{X}_0 , \hat{Y}_b , and \hat{Z}_a are dimensionless aerodynamic coefficients. The aerodynamic coefficients can be obtained by wind tunnel testing or from analytical methods.⁷

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \frac{1}{2} \rho S_{ref} u_0^3 \begin{Bmatrix} 2\hat{X}_0 u \\ \hat{Y}_b v \\ \hat{Z}_a w \end{Bmatrix} \quad (8)$$

Equations (7) and (8) are combined to generate spectral density functions of wind forces on the aerostat. The $S(?)$'s in Equation (7) are substituted for u , v , and w in Equation (8). The remaining constants in Equation (8) are squared resulting in the force spectra in Equation (9). These functions are in units of force squared per frequency band, i.e. newtons squared per radian/second.

$$\begin{Bmatrix} \Phi_u(\mathbf{w}) \\ \Phi_v(\mathbf{w}) \\ \Phi_w(\mathbf{w}) \end{Bmatrix} = \left(\frac{1}{2} \rho S_{ref} u_0^3 \right)^2 \begin{Bmatrix} 4\hat{X}_0^2 S_u(\mathbf{w}) \\ \hat{Y}_b^2 S_v(\mathbf{w}) \\ \hat{Z}_a^2 S_w(\mathbf{w}) \end{Bmatrix} \quad (9)$$

Analysis Model

A very simple FEA model was used—shown in Figure 1. The coordinate system origin is the aerostat nose. Positive directions are X aft, Y port side, and Z downward.

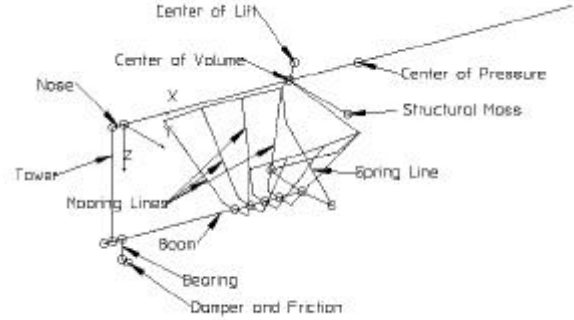


Figure 1 FEA Model

The aerostat was primarily modeled with mass elements and rigid elements. Truss elements were used for the mooring lines, and beam elements were used for the nose structure. A 50-meter aerostat was used for a typical example of modal analysis.

A mobile mooring system was used for the analysis. This type of mooring system does not have a monorail but is attached to the ground at one point. This point is a bearing fixed in all directions except rotation about a vertical axis. The mooring boom and mast are modeled as beam elements. Masses are added to give an appropriate moment of inertia of the rotating structure.

Mass

The mass of the enclosed gasses must be added to the mass of the aerostat structure, payload, and mooring system. In addition to these, apparent mass is added to the aerostat to account for some of the surrounding air moving with the aerostat.

The mooring system moment of inertia is a significant parameter affecting the overall system performance.

The masses used in the example FEA model are given in Table 1. The added mass refers to the mass of the enclosed gasses and the apparent mass.

Damping

Viscous dampers were applied to the centers of pressure of the aerostat to provide aerodynamic damping.

Values of these dampers were calculated from the aerodynamic coefficients and stability derivatives by use of Equation (8) with the u , v , and w terms removed. Rotational damping (roll, pitch, and yaw) were also added in a similar manner.

Table 1 System Parameters

Aerostat length	50 m
Aerostat volume	6180 m ³
Aerostat reference area	337 m ²
Aerostat structural mass	2072 kg
Added mass – axial	3655 kg
Added mass – transverse	9200 kg
Mooring system moment of inertia	2.1E6 kg-m ²

Viscous and coulomb (friction) damping were applied to the mooring system bearing. These are often parameters that can be adjusted to optimize the system.

Stiffness (K matrix)

The aerostat is assumed to be a rigid body attached to the mooring system by an elastic nose structure and elastic mooring lines. The mooring system has an elastic tower and boom section. The mooring system is free to rotate at the base bearing to allow the system to weathervane into the wind.

A spring is attached to the center of pressure of the aerostat to account for weathercocking wind forces. The value of this spring is determined from the wind speed and the aerodynamic coefficients of the aerostat. An alternative is to omit this spring and use a zero frequency rigid body mode.

Typical Results

Results are given for the moored 50-meter aerostat in a 20 meters per second wind with a superimposed turbulence of 1.5 meters per second RMS.

Frequency Analysis

A list of the lowest frequency modes is given in Table 2. A brief description, natural frequency, and modal damping of each mode is given.

The first mode is a lateral weathercocking mode in which the aerostat and mooring system rotate together

about the mooring system bearing. This is shown in Figure 2. The elements showing the envelope and fins are plotting elements for visualization only. The natural frequency of this mode increases with wind speed. It will be a zero frequency or rigid body mode with no wind.

Table 2 Frequency List

Mode Number	Description	Freq. rad/sec	Damping ratio
1	Weathervane	0.58	0.43
2	Lateral	1.1	0.23
3	Axial	2.6	0.12
4	Pitch	3.9	0.036
5	Yaw	6.9	0.29
6	Pitch	8.7	0.14
7	Boom bend	11	0.015
8	Roll & boom twist	14	0.02
9	Boom twist	18	0.002
10	Tower bend	31	0.003

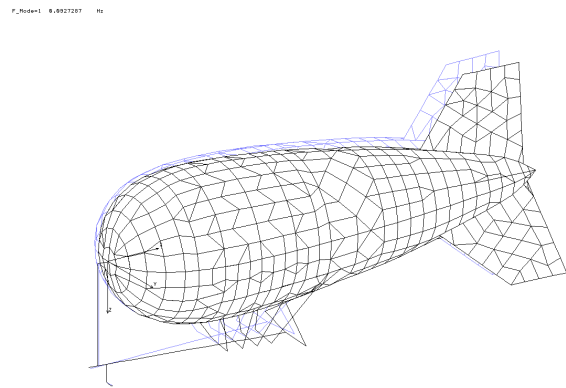


Figure 2 Mode Shape 1 - Weathervane

The second mode is also a lateral mode; however, in this case the aerostat and mooring system move in opposite directions. This mode is potentially troublesome and can cause high lateral nose loads or mooring line loads.

The third and fourth modes are axial in which the nose is compressed or stretched, and the aerostat pitches. See Figure 3.

Mode five is aerostat yaw about the center of mass. The nose deforms laterally for this mode.

Mode six is aerostat pitch about the center of mass. In this mode, the mooring system boom also bends as shown in Figure 4. Each of the higher frequency modes involves primarily mooring system elements. Those modes do not contribute significantly to nose or mooring line forces.

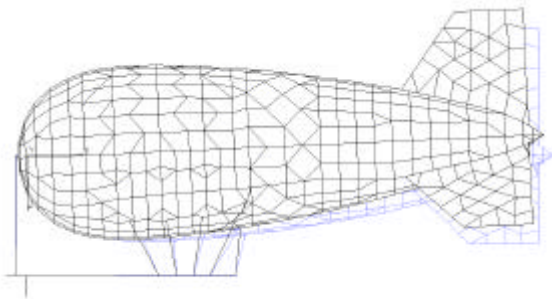


Figure 3 Mode Shape 3 - Pitch

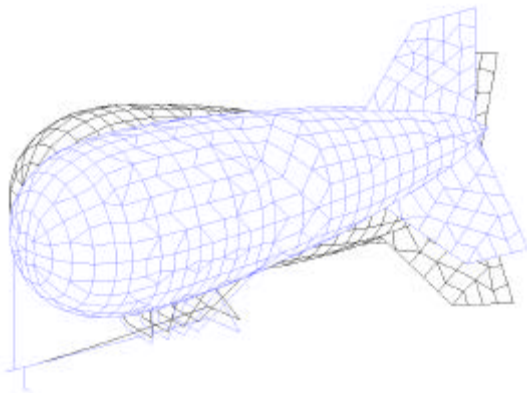


Figure 4 Mode Shape 6 - Pitch

Turbulence Model

The Dryden wind velocity spectral density function is transformed to a force spectral density for the FEA model. The spectrum for each orthogonal direction is shown in Figure 5. The lateral and vertical spectra are

less than the longitudinal spectrum at the lower frequencies. The spectra are nearly flat up to a frequency of 0.1 radians per second. Then the force decreases by one order of magnitude for each magnitude increase in frequency.

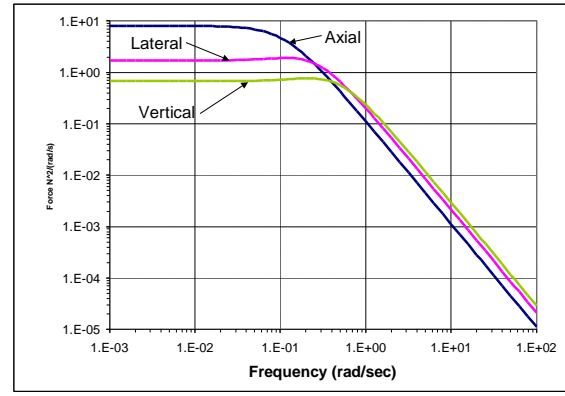


Figure 5 Dryden Turbulence Spectrum

Random Response Analysis

Applying the turbulence spectra to the dynamic model gives spectra of the displacements and forces. The displacement spectrum at the aerostat volumetric center is given in Figure 6. The output spectra of nose forces and line tensions are shown in Figure 7 and Figure 8.

Peaks in the spectra occur at natural frequencies of the system. The amplitudes generally decrease very quickly with increasing frequency. The higher frequency modes do not contribute much to the system forces and deflections.

The deviation or RMS value of a particular spectrum is obtained by integrating over the frequency range. Design values are calculated by adding a factor of the deviation to the static values. Using a factor of three, for example, will give the 99.9% probability design values. On the other hand, if the strength of a particular element is known, the number of deviations to reach that value will give a probability of failure. The slack line limit (zero line tension) can also be determined by dividing the static force by the deviation.

A summary of results is given in

Table 3 Summary of Results. This table gives the static load, the deviation or RMS value, and design value (static plus three deviations) for the critical components.

Conclusions

The method of modal analysis is an excellent method for designing and analyzing a moored aerostat or airship. This can quickly obtain statistical design loads for the nose structure and mooring lines. With existing systems, the allowable wind and turbulence for operation or survival can be assessed.

Parametric studies can be done to optimize certain parameters such as boom offset, mooring system damping, and mooring line locations.

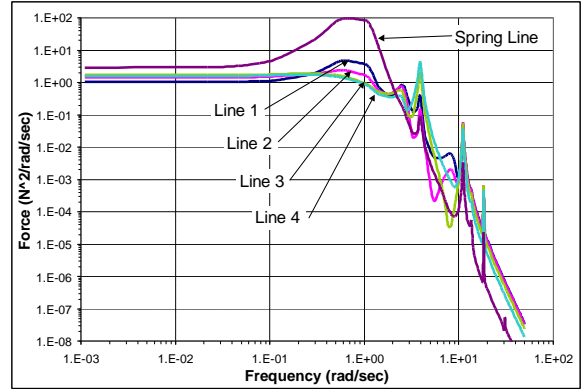


Figure 8 Line Force Spectra

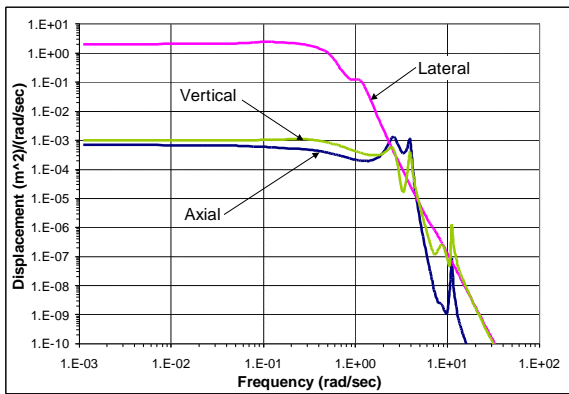


Figure 6 Displacement Spectra

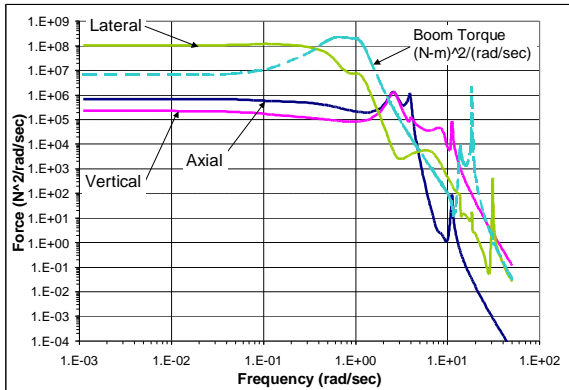


Figure 7 Nose Force and Boom Torque Spectra

Table 3 Summary of Results

Component	Static	RMS	Design
Nose Axial Force	-1593	1488	2871
Nose Vertical	4494	1262	8280
Nose Lateral	0	7479	22437
Mooring Line 1	1526	1612	6362
Mooring Line 2	1738	1279	5575
Mooring Line 3	1790	1287	5651
Mooring Line 4	1616	1411	5849
Spring Line	203	2581	7946
Boom Torque	0	13620	40860

Table 3 notes: SI Units, Static forces are for a 20 m/s wind, RMS is the deviation due to a 1.5 m/s RMS turbulence, and Design is the Static plus 3.0 times the RMS value

References

- ¹ Jones, S.P., *Nonlinear Dynamic Simulation of a Moored Aerostat*, AIAA Paper 87-2505, AIAA 7th Lighter-Than-Air Technology Conference, Monterey, CA, August 17-19, 1987.
- ² Redd, L.T., Bland, S.R., and Bennett, R.M., *Stability Analysis and Trend Study of a Balloon Tethered in a Wind, with Experimental Comparisons*, NASA TND-7272, Oct. 1973
- ³ DeLaurier, J.D., *A Stability Analysis for Tethered Aerodynamically Shaped Balloons*, Journal of Aircraft, Vol. 9, Sept. 1972, pp. 646-651
- ⁴ Jones, S.P., Krausman, J.A., and Sunkara, B.B., *Dynamics of the Stars Aerostat*, AIAA paper No. 83-1990, AIAA Lighter-Than-Air Systems Conference, Anaheim, California, July 1983
- ⁵ NASA TM 4511, *Terrestrial Environment (Climatic) Criteria Guidelines for Use in Aerospace Vehicle Development*, 1993 Revision
- ⁶ MIL-F-8785C, *Flying Qualities of Piloted Airplanes*, September 24, 1991
- ⁷ Jones, S.P. and DeLaurier, J.D., *Aerodynamic Estimation Techniques for Aerostats and Airships*, Journal of Aircraft, Vol. 20, February 1983