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**Tethered Aerostat Performance Modeling**
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# Tethered Aerostat Performance Modeling

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## Abstract

This paper presents data and analy-
sis relative to a simple model of
buoyant lift which is considered to be
due to an independent parcel of helium,
free to expand by expelling weightless
air from a ballonet. The model of the
atmosphere is derived from the hydro-
static equation, assuming a constant
lapse rate and boundary conditions at
the surface. Corrections to buoyancy
include those for superpressure, super-
heat and humidity, for which equations
and nomographs are given. Experimental
superheat data, spanning two days of
calm conditions including both day and
night helium and air temperature, are
presented. These and other data are the
basis for semiempirical curves showing
superheat as a function of wind in the
daytime and at night. A nomograph gives
the effect on lift of both air and
helium dew points. The "optimum" alti-
tude and helium fill are derived from
the intersection of "vent" ceiling when
the ballonet is empty and "lift limit"
curves representing the gross weight,
free lift and lift loss due to super-
heat. Optimum altitude curves for the
TCOM 31-meter aerostat are included.

## Nomenclature

$a$ = atmospheric lapse rate or negative temperature gradient
$a_0$ = standard lapse rate
$C_f$ = coefficient in the Nusselt number equation for forced convection
$C_n$ = coefficient in the Nusselt number equation for free or natural convection
$Gr$ = Grashof number
$H$ = altitude
$k$ = exponent in the Nusselt number equation for natural convection
$m$ = exponent in the Nusselt number equation for forced convection
$n$ = nondimensional exponent defined in Eq. (5)
$Nu_\ell$ = Nusselt number based upon hull length
$p$ = ambient atmospheric pressure
$p_0$ = standard pressure
$p_s$ = barometric pressure at the surface
$Pr$ = Prandtl number
$Re$ = Reynolds number
$R_h$ = relative humidity (%)
$T$ = absolute air temperature
$T_{dp}$ = absolute dew point temperature ($^\circ$R)

$T_s$ = absolute temperature at the surface
$T_g$ = absolute temperature of the enclosed gases
$T_0$ = standard temperature
$V_t$ = total volume of the enclosed gases
$\Delta L_p$ = lift differential due to super-pressure
$\Delta L_t$ = lift differential due to super-heat
$\Delta p$ = pressure differential of the internal gases over ambient air (superpressure)
$\Delta T$ = temperature differential of the internal gases over the ambient (superheat)
$\rho$ = specific weight of air
$\rho_s$ = specific weight of air at the surface
$\rho_0$ = specific weight of air under standard conditions

## Introduction

The dynamics of tethered aerostats
has been given considerable attention in
recent years. Surprisingly, however,
little has been written about steady-
state performance, even though that is
usually the subject of greatest interest
to the user. In fact, there is a
shortage of material for training and
reference, covering the more elementary
aspects of tethered aerostat performance
and modeling. Available references in-
clude Brown and Speed[1], Myers[2],
Wright[3] and Jones[4].

With increased utilization of large
tethered aerostats and improved instru-
mentation and telemetry, more is being
learned about the factors affecting
static performance. Increasing demands
on the capabilities have required more
detailed analysis for improved perform-
ance. Of equal importance to modeling
and analysis are methods of presenting
the operating envelope in a meaningful
way to users who may not be trained or
inclined toward analysis. This paper
presents some of the concepts and rela-
tionships useful in modeling and
predicting steady-state performance of
tethered aerostats. Emphasis is placed
on the factors which affect buoyant lift
and the computation of optimum helium
fill and operating altitude.

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## Model of the Atmosphere

An essential foundation for performance prediction is an accurate model of the atmosphere. The effect of ambient conditions is so strong that it is certainly not sufficient merely to use U. S. Standard Atmosphere tables[5]. A clear and concise derivation of the relationships of temperature, pressure, air density (or specific weight ) and altitude is given by Durand[6]. With the realistic assumption of a constant lapse rate, these relationships flow directly from the hydrostatic equation with the surface conditions as a boundary.

$$dp = -\rho dH \qquad (1)$$

$$T = T_s - aH \qquad (2)$$

Integration yields

$$p = p_s \left[ \frac{T}{T_s} \right]^n \qquad (3)$$

$$\rho = \rho_s \left[ \frac{T}{T_s} \right]^{n-1} \qquad (4)$$

where

$$n = \frac{\rho_0 T_0}{p_0 a} \qquad (5)$$

For the standard lapse rate, the value of $n$ is determined.

$$a_o = .0065°C/m \doteq .0035662°F/ft$$

$$n_o = 5.2558$$

If the surface is at sea level, then $H$ is the geopotential altitude which, for practical purposes, is the same as the geometric altitude in the lower troposphere.

The presumption is dry air. The effect of humidity, which reduces the air density, can be accounted for by the use of virtual temperature[7]. This is the temperature of dry air which would have the same density as the moist air at the same pressure. More will be said later about the effect on buoyancy of water vapor in both the air and the helium.

## Buoyancy Model

For dynamic analysis the Archimedean buoyancy of the entire body and the total mass, including the enclosed gases, must be considered. Due to the variable volume of air in the ballonet however, the weight and center of gravity are not constant but depend upon air volume and shape. The ballonet air, being heavier than helium, forms a level interface with it, as illustrated in Figure 1. The mass center of the air is therefore dependent not only on the volume, but also the pitch angle. For each aerostat design, a family of curves must be developed giving the air mass center as a function of volume and pitch.
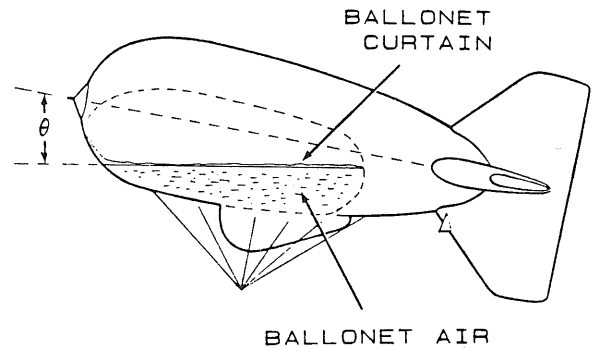


Fig. 1   Leveling of air in the ballonet of a tethered aerostat.

A simplified model, commonly used for steady-state analysis of aerostat performance, envisions the lifting gases (helium) as free to expand by expelling neutrally-buoyant air from the ballonet. Thus, the gross buoyant lift is that of an independent parcel of helium. Neglecting the small pressure differential (superpressure) and with no temperature differential relative to ambient (superheat), the lift of such a parcel is independent of temperature and pressure and therefore of altitude. This model has the added advantage that the gross weight is that of the structure alone, which is constant with a fixed center of gravity. The variable is the center of buoyancy, the centroid of the helium volume, which is a function of its size and shape, obtained directly from the ballonet air mass curves.

This simple model requires corrections for deviations from the simplifying assumption that the lifting gas is at the same temperature and pressure as the surrounding gas and that both are dry. The gross lift of the helium thus corrected is referred to as the Standard Gross Lift.

## Correction for Superpressure

The superpressure is the differential of pressure inside the hull to the ambient and is responsible for main-

taining the rigidity of the hull. Common practice is to maintain about two inches of water gauge over the dynamic pressure. This compression of the enclosed gases results in a loss of lift given by

$$\Delta L_p = -\rho V_t \frac{\Delta p \, T}{p \, T_g} \tag{6}$$

which reduces to

$$\Delta L_p = -\frac{\rho_o T_o V_t}{p_o} \frac{\Delta p}{T_g} \tag{7}$$

Since the absolute temperature does not vary greatly, the lift loss is almost independent of altitude. For the large TCOM 365 aerostat, having a total volume including fins of 421,000 cu ft, compression to 2 IWG results in a lift loss of about 160 pounds. To this must be added the dynamic pressure, which at 50 kts is about an additional 1.5 IWG, corresponding to 120 lbs lift loss. The latter will be offset by aerodynamic lift, but must be taken into account in analysis.

On large aerostats the vertical gradient of pressure due to the helium head is significant, being about 3/4 IWG over the 60 ft diameter of the TCOM 365 aerostat. Therefore, care must be taken as to the location of the pressure transducer relative to the average superpressure.

## Correction for Superheat

The term "superheat" in common usage refers to the temperature differential of the internal gases to the ambient air (supertemperature). It results from the radiation environment and is usually positive in the daytime due to solar radiation and negative at night when the aerostat radiates to a clear sky. The effect is maximum during clear, calm conditions when the heat exchange with the ambient air is by natural convection. Relative wind diminishes superheat, due to forced convective heat transfer.

Superheat has two effects. It increases the volume of helium, expelling air from the ballonet, and increases the lift by decreasing the density of the enclosed gases. Assuming the enclosed gases are at a uniform temperature, the net effect on buoyant lift is analogous to that for superpressure.

$$\Delta L_t = \rho V_t \frac{\Delta T}{T_g} \tag{8}$$

For the TCOM 365 aerostat at 10,000 ft on a standard day, this amounts to 49 lb per °F superheat.

The magnitude of superheat depends upon many factors including the hull material, time of day, cloud cover, season and relative wind. Figure 2 presents data taken on a TCOM 250 aerostat at Grand Bahama Island during an unusual period of clear, calm conditions for two days in January 1974. The wind velocity was less than 5 knots and the aerostat's altitude was maintained between 8,500 and 10,800 ft. The shielded temperature probe was hanging about 15 ft from the top of the hull at the major diameter. Like all subsequent TCOM aerostats, the hull material included an outer layer of white, Tedlar film.
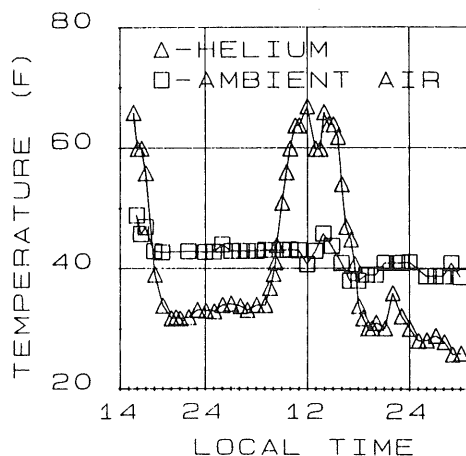


Fig. 2  Helium and air temperature
        on a TCOM 250 aerostat
        during calm conditions,
        Grand Bahama Island,
        January 18-20, 1974.

During the day, subjected to solar radiation, the helium temperature reached about 25°F higher than the ambient air, while at night, due to radiation to space, the differential was negative by more than 10°F. These results are fairly typical of observations with other aerostats under similar conditions, irrespective of location and season.

Based upon empirical data and heat transfer theory, the curves in Figures 3 and 4 have been derived to estimate the superheat for day and night conditions, respectively. For these curves the net heat flux was estimated from empirical data. At low winds, free convection is assumed, where the Nusselt number is given by

$$Nu_\ell = C_n \, (GrPr)^k \tag{9}$$

With k taken to be 1/3, as for large horizontal cylinders[8], the heat transfer coefficient and the superheat are independent of aerostat size.

At higher winds, where forced convection predominates

$$Nu_\ell = C_f Re^m \qquad (10)$$

Estimated from friction drag data, the exponent, m, is very near unity where the superheat is again independent of size. Using McAdams' Rule[8], the mechanism which gives the highest film heat transfer coefficient or lowest absolute superheat is assumed.
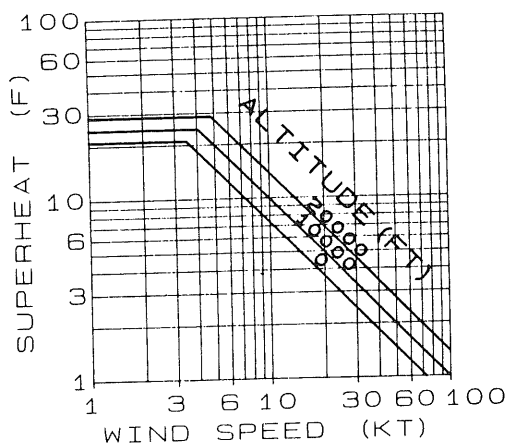


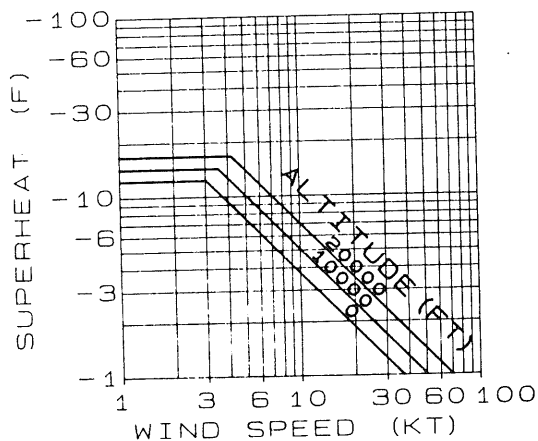Fig. 3  Semiempirical curves for estimating day superheat.



Fig. 4  Semiempirical curves for estimating negative super-heat at night.

Effect of Water Vapor

Water vapor is lighter than air so that moist air is less buoyant than dry air. This may be offset, however, by water vapor inside the aerostat. A little-known fact is that, although relatively impermeable to helium, most aerostat hull materials transmit water vapor rather well. Thus, tests on TCOM laminated hull material using ASTM E-398 gave a water vapor transmissivity of 1.3 gms/100 in² - 24 hr at 100°F. This is over 10 times the helium permeability measured at the same temperature[9], when compared on the basis of volume transmitted per atmosphere.

While moisture in the ambient air decreases lift, water vapor inside the hull, being lighter than air, increases lift. The water vapor concentration inside the hull will approach that of the outside air, in which case there will be no net effect on lift. For that reason and the difficulty of measuring the humidity in the helium, this effect is usually ignored. However, ambient humidity fluctuates widely and that inside the aerostat lags by many hours so that the concentrations are seldom the same. Humidity should be taken into account when making precise measurements such as lift loss rate.

Figure 5 shows the effect of water vapor both in the air and the helium on the gross lift factor or ratio to the lift of dry helium in dry air. The curves are given in terms of the dew point which is related to the temperature and relative humidity by the approximate formula

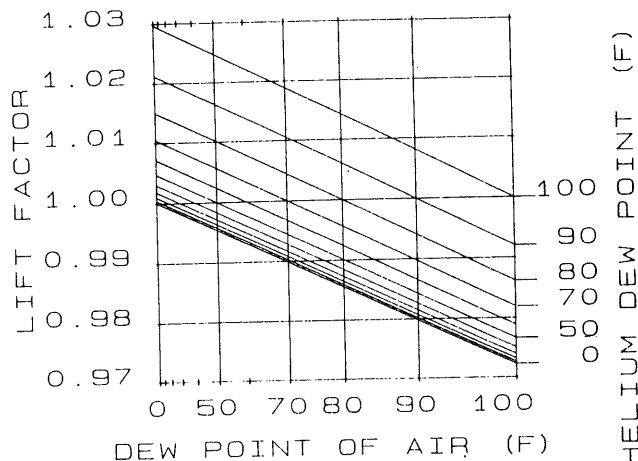$$\frac{1}{T_{dp}} = \frac{1}{T} - \frac{\ell n \ (R_h/100)}{9540} \qquad (11)$$



Fig. 5  Nomograph showing the effect of water vapor on helium gross lift.

In addition to the effect on buoyant lift, water vapor in the helium affects the ceiling altitude of the aerostat. Since it occupies volume, the

altitude at which the ballonet is empty (vent ceiling) will be lowered. If the temperature reaches the helium dew point, the water vapor, which contributes to lift, will be converted to liquid water ballast, reducing the free lift. On large aerostats it may be necessary to drain accumulated water from time to time.

## Optimum Altitude

It is essential to know how high an aerostat and payload can safely fly under calm conditions, and what helium fill must be provided. The operating altitude is determined by two factors, illustrated in Fig. 6, which is a plot of altitude as a function of helium fill in terms of gross lift. The curves of positive slope represent the lift limit for the gross weight, including tether, and free lift. The higher the altitude, the greater the tether weight and thus the more lift required. The slope is the unit weight of the tether. At night, negative superheat reduces lift, as shown by the displaced curve.

SUPERHEAT (F)

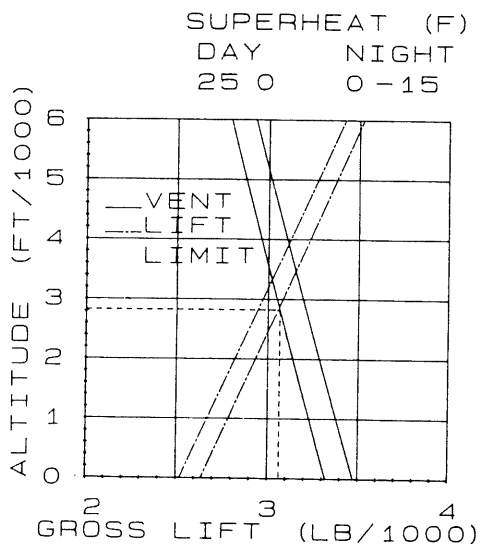| DAY | NIGHT |
|-----|-------|
| 25 0 | 0 -15 |

Fig. 6  Vent altitude and lift limit curves for determining optimum altitude and lift (dashed lines).

The second factor determining operating altitude is the "vent" ceiling, represented by the curves of negative slope. This is the altitude at which the helium has expanded to fill the available volume, expelling all the air from the ballonet. Any further increase in altitude would cause an increase in hull pressure which, on large aerostats, results in the venting of helium. These curves are very sensitive to superheat, which reduces the vent altitude.

The intersection of the lift limit and vent curve gives the altitude that can be maintained both day and night with a minimum specified free lift, and allowances for superheat. This corresponds to an optimum helium fill. Of course, as pointed out earlier, wind reduces superheat and it will be possible to fly higher than the optimum altitude under some conditions, but that will be at the expense of free lift based on helium buoyancy. For flights of long duration it may be necessary to compromise altitude by overfilling to account for helium leakage.

Figure 6 is specific for a given set of ambient conditions. In particular, the ambient temperature and pressure will affect the vent curves. At lower temperatures the aerostat can contain a greater mass of helium, providing more lift. Figure 7 shows how the optimum altitude varies with sea-level temperature for the TCOM 31-meter aerostat, with payload weight as a parameter. Standard sea-level barometric pressure and lapse rate are assumed. The free lift is 20% of the gross lift and the daytime superheat is taken to be 25°F. In these curves, negative superheat at night is not taken into account. It is a matter of strategy to permit negative superheat, which only exists under calm conditions, to be deducted from the static free lift.
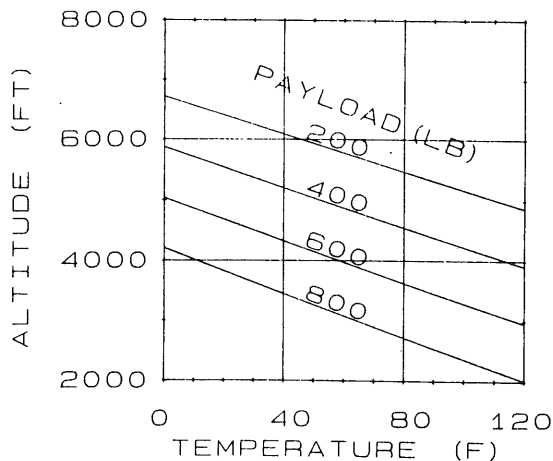
Fig. 7  Optimum altitude as a function of sea-level temperature for the TCOM 31 meter aerostat with various payloads. Free lift is 20% of the gross lift.

## Summary

Some of the data, analyses and
concepts used in the prediction of
tethered aerostat steady-state perform-
ance have been presented. Included is a
model of the atmosphere derived from the
hydrostatic equation with the assumption
of a constant lapse rate and boundary
conditions at ground level.

A simplified model of buoyancy
envisions the gross buoyant lift as due
to an independent parcel of helium which
is free to expand by expelling air from
the ballonet. The latter is assumed to
be weightless and the gross weight that
of the structure alone. For this model
the buoyant lift is independent of tem-
perature and pressure so long as there
is no differential with the ambient air.
However, corrections must be made for
superpressure, superheat and water vapor
content of the helium and the air.

Based upon heat transfer theory and
empirical data, some of which are
presented, curves for the estimation of
day and night superheat have been
derived. A nomograph has been developed
for estimating the effect of water vapor
on gross lift.

"Optimum altitude" is defined as
the altitude at which the aerostat with
a given payload and free lift can fly
both day and night with allowances for
maximum superheat. This corresponds to
an "optimum" helium fill and is depend-
ent upon sea level temperature. Optimum
altitude curves for the TCOM 31-meter
aerostat for various payloads were pre-
sented.

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