Characterizing the Shape of LTA Vehicles Using Photo-grammetry and Stretch Functions

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After inflating a flexible airship or aerostat envelope, the shape may change from the theoretical design for various reasons. Knowledge of the inflated shape is important to obtain accurate volumetric properties and for hull attachments such as aerostat racks, gondolas, and rigging. The actual inflated shapes of lighter than air (LTA) vehicles have traditionally been determined using measuring tapes and plumb bobs to measure the projected profile and surface lengths. A method of characterizing this shape more accurately and completely has been developed using photogrammetry and stretch functions. Photogrammetry is also being used during diaphragm tests to measure the shape and material strain.

Nomenclature

f	=	stretch function
g	=	gore length
h	=	distance from gore centerline
k	=	radius scale factor
L	=	overall length
m, n	=	shape coefficients
q	=	Fourier series coefficients for squeeze
r	=	mean radius of cross section
s	=	seam length
x	=	axial length
y	=	distance toward the port side of centerline
z	=	distance down from centerline
ΔL	=	length offset
Φ	=	cylindrical angle from top of hull

Subscripts

0	=	theoretical pattern dimensions
1	=	stretched or reference shape dimensions
b	=	lateral deformation due to bend
h	=	vertical deformation due to hump
s	=	stretched dimension (for length, radius or squeeze)
t	=	deformation due to twist

Note: Unless otherwise indicated, all values are non-dimensional, normalized by the overall length.

I. Introduction

AEROSTAT hulls are usually bodies of revolution using various shape functions such as the Upton family of equations as given in Eq. (1) , where r_0 is the radius, x_0 is the axial length from the nose, L is the overall length,

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ΔL is the length offset, and k, n, and m are shape constants. See Figure 1 for coordinate system definition. Subscript 0 denotes pattern dimensions.

$$r_0 = k(x_0 + \Delta L)^n (L - x_0 - \Delta L)^m \tag{1}$$

The aerostat hull is made from flat patterned two-dimensional segments known as gores (Figure 2). The gore half width, **h**, is the circumference divided by the number of gores divided by 2. The gore centerline length, **g**, can be calculated from the three dimensional shape using the standard arc-length formula.

The flat gore seam, **s**, is obviously longer than the centerline length. When the assembly of flat pattern gores is inflated, the centerline length and the seam length (and all meridian lengths) must deform to have the same length in order to have a smooth non-faceted shape. Due to this 2D to 3D process, material properties, and the manufacturing process, the actual pressurized shape will be different from the pattern shape. Further, after lifting gas and weights are added, the shape changes become considerably more complex.

The stretched shape of an LTA vehicle is very difficult to calculate accurately. Therefore, it is necessary to measure the hull of the vehicle.

II. Shape Measurements

The traditional method of aerostat measurement was by the use of measuring tapes and plumb bobs. Other measurement methods, such as the use of global positioning systems, laser trackers, and laser range finders, have had very limited success. This paper concentrates on the use of photogrammetry for shape measurements. Photogrammetry has been used for obtaining correlation with non-linear structural analysis of super pressure balloons[1].

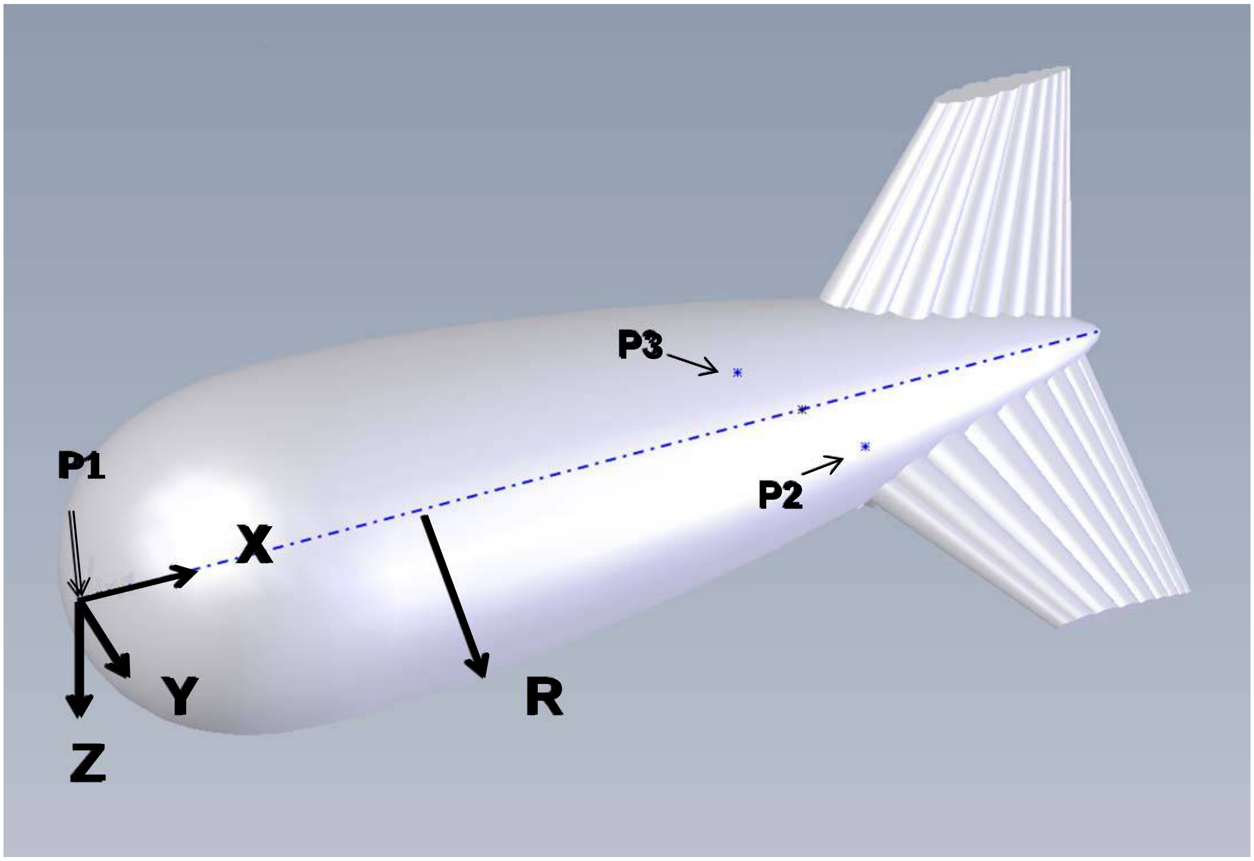

Figure 1 Aerostat Coordinate System and Reference Points

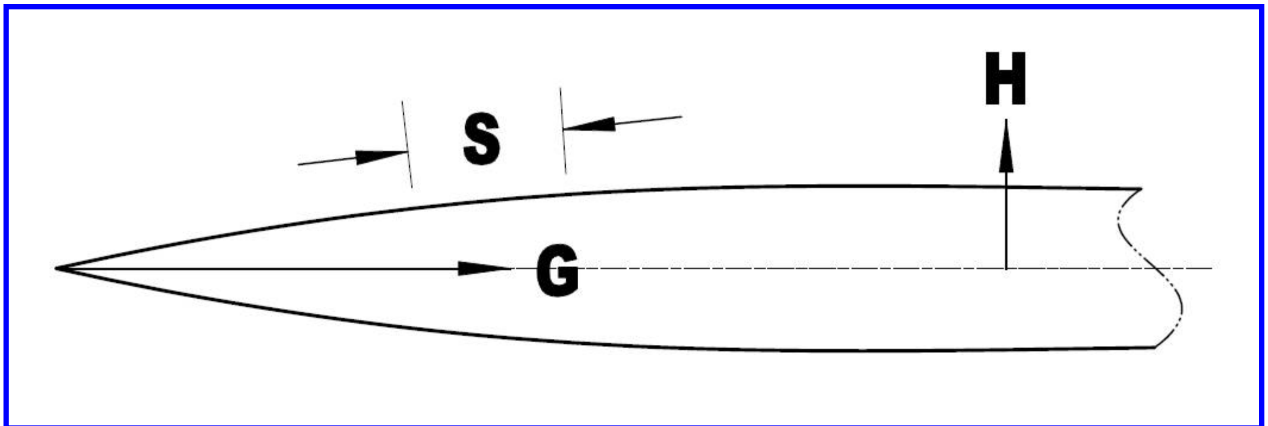

Figure 2 Flat Pattern Gore

A. Traditional Measuring Tape and Plumb Bob Method

For any measurement method (measuring tapes or photogrammetry), the aerostat must be prepared with markings on the outside to show locations of points of interest. If references for cutting and manufacture are marked on the inside, then they must be transferred to the outside before inflation. Otherwise, the measurements will be limited to identifiable points such as load patches or controlled seams.

During the test, it is important to maintain uniform conditions such as hull pressure. Otherwise, the shape will be changing while being measured.

The circumference is measured at several stations from top to bottom and compared to the pattern circumference to obtain the radial strain. The length can also be measured along longitudinal seams to obtain the stretched gore length.

The aerostat is leveled by holding with close haul lines and a nose line such that the vertical distance at points P1, P2, and P3, (see Figure 1) are equal. P2 and P3 are points on the horizontal seam just forward of the fins. With the aerostat level, the horizontal seam points are projected to the floor using plumb bobs. The height from the floor to the horizontal seam is also measured at these points. From this profile, the stretched axial length can be determined. The twist of the hull, the hump (vertical deformation from a straight line), and bend (horizontal deformation) can be calculated.

It is very difficult to hold the aerostat still during these measurements (which can take several days to complete). If additional constraints are used, these in themselves cause unwanted deformations. The conditions of temperature and lift can also change. These factors limit the accuracy of this method for shape measurement. More accuracy also requires more plumb bobs, but then the difficulty of making more measurements of the moving aerostat also increases.

B. Photogrammetry Measurements

Photogrammetry is the science of making measurements from photographs. The fundamental principle used is triangulation. By taking photographs from several locations, the 3-dimensional coordinates of an object can be determined from 2-dimensional photographs.

Preparation was made by transferring marks to the outside of the hull and attaching retro-reflective targets at specified pattern locations. Approximately 500 targets were used at the gore seams on the upper surface of the hull. Since photogrammetric measurements are inherently dimensionless, scale bars were attached to the aerostat for scaling to the actual coordinates

Approximately 30 high-resolution digital cameras were mounted in various locations from directly overhead on the hangar catwalk to tripods on the floor. The intermediate camera locations were on the hangar ceiling structure. The single lens reflex cameras were calibrated and equipped with ring flash and remote triggers. All cameras were triggered within one second; however, the timings were slightly staggered to prevent cross interference with the flashes.

The computer software PhotoModeler® [2] was used both to analyze the photographs, creating a point cloud output, and to calibrate the cameras used in the test setup. Camera calibration involved taking multiple pictures of a test grid from multiple angles. The program then used the pictures to build an accurate model of that camera and its optics. These calibration files were then applied to each of the pictures taken for the test case to create a more accurate output.

While photogrammetry is a process that involves a significant amount of compromise, the end results are better than traditional methods.. Ideally, each target should have three or more cameras seeing it. Each camera should see the target from as close to 90° relative to the other cameras as possible (imagine 1 camera on each axis of a 3D graph aimed at the origin). There should be a fair amount of overlap between the pictures, allowing adjacent pictures to be stitched together with a high degree of confidence. Ideally, all cameras would be equidistant from the target, allowing an ideal target size and ideal camera settings to be determined. Other constraints include the number of cameras available, the geometry of the aerostat (cameras may not be able to see certain targets), and also the availability of places to hang the cameras.

To help with optimizing all of these variables, TCOM contracted an advisor from the Virginia Military Institute, Joseph R. Blandino[3]. Using software designed to optimize complex photogrammetry setups, he was able to recommend camera coordinates as well as camera pitch, roll, and azimuth. For more information on photogrammetry setup and optimization, see Ref. 4 and Ref. 5.

Note, PhotoModeler does not require any prior knowledge of camera locations. The cameras can be anywhere, and as long as the program has enough points and enough overlap between pictures it can solve for the camera locations.

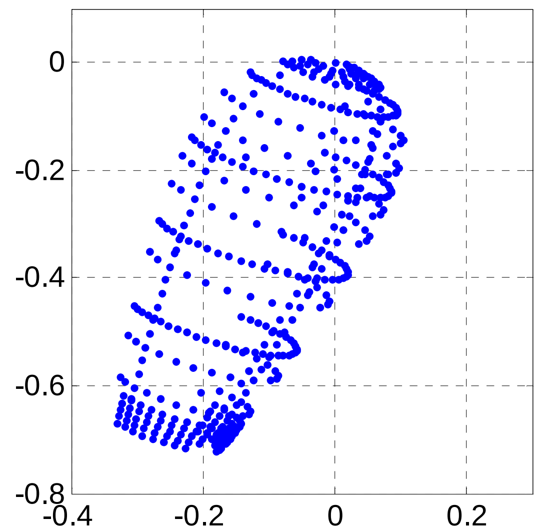

Figure 3 Output from Photogrammetry Software
This plot is in an arbitrary coordinate system.

C. Data Reduction and Analysis

The point cloud from PhotoModeler comprises the 3-dimensional coordinates of each target in some arbitrary coordinate system. The software also calculates the precision of each coordinate. A plot of a typical output is shown in Figure 3. By examining these plots in a spreadsheet, three points are selected to define a body fixed coordinate system. Plots of the points in the body fixed coordinate system are shown in Figure 4 and Figure 5. The coordinates of reference points P1 is (0,0,0), $Z_{P2} \equiv Z_{P3} \equiv 0$, and $Y_{P2} \equiv -Y_{P3}$.

The measured points must be associated with the set of pattern dimension points. This is done by finding the closest pattern point to each of the measured points. Since the targets are three dimensional, corrections are made to account for the offsets from the surface. Once this is done, the stretch of each point is calculated.

D. Photogrammetry Accuracy

Precision is defined to be the repeatability of the measurements. The precision calculated by the computer software for each point was from 4 to 8 mm (mean plus one standard deviation). The precision depends on the number of cameras and their relative position.

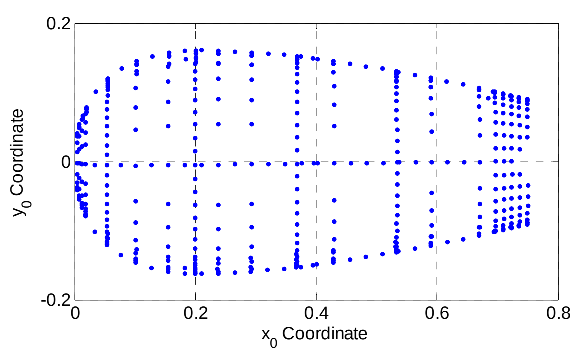

Figure 4 Top View of Photogrammetry Points

The absolute accuracy of photogrammetry depends on a number of factors: camera resolution, camera calibration method, angles between photos, photo orientation quality, photo redundancy, the quality of the targets, and the size and accuracy of the scale bar. The equipment and setup for our test met most of the criteria for “average accuracy” as defined by PhotoModeler, which is 1 part in 5000. For example, a 50m long object would have an absolute accuracy of 1 cm. We also checked the accuracy by using multiple scale bars and by cross checking selected points with measuring tapes.

E. Reference Shape Characterization

The stretched shape of an aerostat taken under nominal conditions of pressure, weight, and lift, is referred to as *the reference shape*.

Instead of interpolating from a look-up table or developing a single curved fitted function, the reference shape is in the form of several *stretch functions.* These stretch functions are as follows:

- Circumferential (radial) strain, f_{rs}
- Axial stretch at centerline, f_{xs}
- Bend (horizontal deviation of longitudinal axis from straight line in the Y direction), f_{yb}
- Hump (vertical deviation of longitudinal axis from straight line in the Z direction), f_{zh}
- Twist of the longitudinal axis, $f_{\Phi t}$
- Squeeze or out of roundedness, f_q

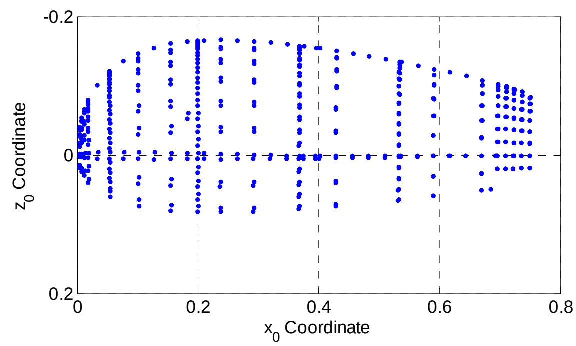

Figure 5 Side View of Photogrammetry Points

The xyz coordinates of any point can be transformed from pattern dimensions to reference shape dimensions by using these stretch functions.

1. *Horizontal Seam Functions*

Whether measurements are made by photogrammetry or by traditional methods, four of the six stretch functions can be obtained from the horizontal seam data only. With the traditional measurement method, the horizontal seam is projected to the floor using plumb bobs. These stretch functions are in the form of polynomials or rational functions.

2. *Functions Requiring Additional Points*

Sets of points that have the same axial pattern length are selected for calculating the circumferential strain and squeeze functions. At each applicable X station, points are projected onto a plane fitted in the least squares sense. To find circumference, the length of a spline that is passed through each set of points is calculated. The circumferential strain is taken to be the strain of the mean radius of the hull. The circumference can also be measured accurately with a measuring tape.

The cross section of the actual hull deviates from the theoretical circular cross section and is non-circular: The width of the hull is smaller than the nominal diameter calculated by the circumference divided by 2π. This *squeeze* is caused by buoyant lift, which is the result of the internal pressure varying with height in a *hydrostatic* sense.

The squeeze function is more complex than the other stretch functions because it varies not only with X location, but also with Y and Z. A good way to represent this function is with a cosine and sine series, Eq. (2).

Here, q_s is an array of coefficients, unique for each X location, and Φ_0 is the cylindrical angle of the pattern point (X_0, Y_0, Z_0) when expressed in cylindrical coordinates.

$$\begin{Bmatrix} z_q \\ y_q \end{Bmatrix} = \begin{bmatrix} 1 & \cos(\phi_0) & \cos(2\phi_0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin(\phi_0) & \sin(2\phi_0) \end{bmatrix} \{q_s\} \qquad (2)$$

3. *Combining All of the Stretch Functions*

By combining the six stretch functions, the reference shape coordinates for any point on the hull surface can be calculated by Eq. (3). As mentioned previously, $\mathbf{x_1}$, $\mathbf{y_1}$, and $\mathbf{z_1}$ denote dimensions determined by factory measurement. In evaluating $\mathbf{x_1}$, it is assumed that a set of points at a given $\mathbf{x}$ station location remain in a plane. This plane is rotated by the bend and hump angles Φ_b and Φ_h. $\mathbf{x_s}$ is the stretched length measured along the longitudinal axis. Note that $\mathbf{y_1}$ and $\mathbf{z_1}$ must be evaluated before evaluating $\mathbf{x_1}$.

$$\begin{Bmatrix} y_1 \\ z_1 \end{Bmatrix} = \begin{Bmatrix} y_b \\ z_h \end{Bmatrix} + \begin{bmatrix} \cos(\phi_t) & \sin(\phi_t) \\ -\sin(\phi_t) & \cos(\phi_t) \end{bmatrix} \begin{Bmatrix} y_q \\ z_q \end{Bmatrix}$$
$$x_1 = x_s + y_1 \sin(\phi_b) + z_1 \sin(\phi_h) \tag{3}$$

III. Comparison of Results From Different Aerostats

The reference shape has been measured using photogrammetry on several aerostats. Each aerostat hull was built from the same patterns and used the same fabric specification; however, each one of them showed shape differences. These differences are primarily the result of variation in the actual elastic properties of the specific material lots used, variations in the manufacturing process, the pressure history of the aerostat, the actual lift and weight distribution, and the weather conditions at the time of the measurements.

A. Radial stretch

The radial stretch for each serial number was about seven percent as shown in Figure 6. The degree of radial stretch is important for calculating an accurate volume, and for fitting of attachments such as aerostat racks.

B. Axial Stretch

The mid points of diametrically opposite points along the horizontal seam are used to calculate axial strain, bend, and hump.

Notice in Figure 7 that there is considerable variation in axial length. Axial stretch is caused both by material stretch and by geometry change with pressurization. Most aerostats became longer with stretch, but some actually become shorter.

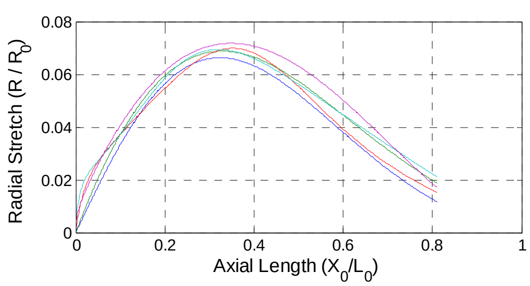

Figure 6 Radial Strain Comparison

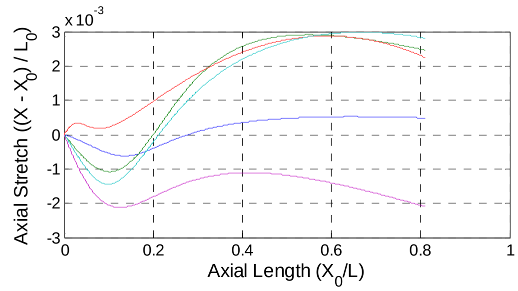

Figure 7 Axial Stretch Comparison

C. Hump and Bend

Hump is caused primarily by the variation in shear and bending moment along the axis due to lift and weight. Material shear modulus can also affect the degree of hump. In Figure 8, you can see that there was considerable variation from one serial number to the next. In all cases, the highest degree of hump is near the center because of the influence of the confluence lines and weight of attached components.

The degree of bend was very small, an order of magnitude smaller than hump.

D. Twist

The variation in the twist angle is shown in Figure 9. This varied from negligible to several degrees over the length of the aerostat. Twist, caused primarily by material bias, should be minimized for proper fit of rigging and for proper fin alignment.

E. Squeeze

The aerostat did become narrower and taller than the circular cross section. The degree of squeeze was more pronounced near the center of the hull.

F. Sensitivity of Shape to Loading Conditions

Some additional photogrammetry shape measurements were made with different conditions of pressure, lift, and pitch angle.

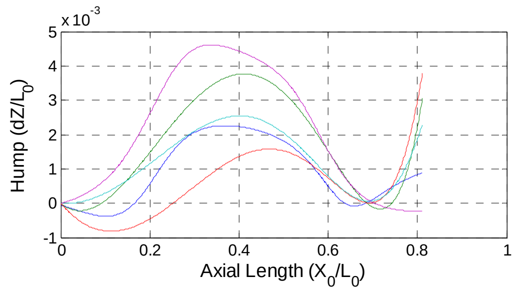

Figure 8 Hump Comparison

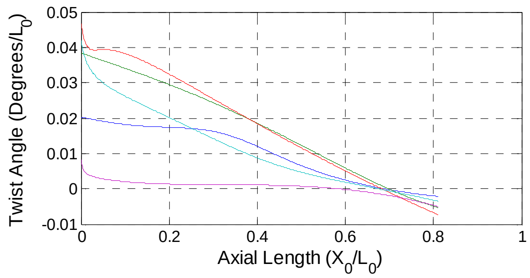

Figure 9 Twist Angle Comparison

IV. Introductions to Fabric Strain Measurement by Photogrammetry

Load patches are placed on aerostat hulls to transfer loads from the rigging to the hull fabric. These load patch assemblies are tested on inflated diaphragms to find the failure load and mode of failure. Photogrammetry is also used during diaphragm tests to measure the shape and material strain.

In these diaphragm tests, 18 foot diameter diaphragms are fitted with a load patch, pressurized to obtain operational conditions of stress, and the load patch is pulled until destruction. The measured strains during the tests are compared to finite element analysis of the same loading configuration.

A typical load patch test is shown in Figure 10. The load patch is bonded to the inside surface of the diaphragm fabric. The webbing comes through a slot in the fabric and is attached to a tensioning device off to the left. The dark sports on the diaphragm are photogrammetry targets. A typical three component set of strain plots is shown in Figure 11.

Figure 10 Diaphragm Tests of Load Patch

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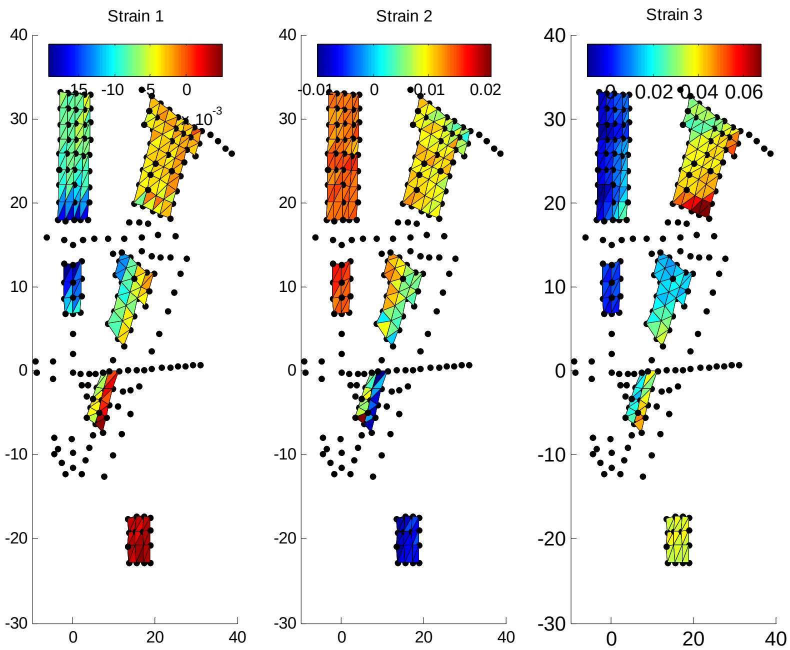

Figure 11 Fabric Strain During Load Patch Test
Strain components are 1= horizontal, 2=vertical (aligned with load direction), and 3=shear.

V. Conclusion

Photogrammetry is a viable and accurate method of measuring aerostat hulls. Photogrammetry overcomes some of the shortfalls of the traditional method that uses measuring tapes and plumb bobs. Photogrammetry is much more accurate and also provides the complete three dimensional shape.

VI. References

[1] Welch, J. V., Wang, S., Blandino, J., and McEvoy, K., "Super Pressure Balloon Non-linear Structural Analysis and Correlation Using Photogrammetric Measurements", AIAA 2007-7447, *AIAA 5th Aviation, Technology, Integration, and Operations Conference*, Arlington Va. 2005.

[2] PhotoModeler® v6, Eos Systems Inc, Vancouver BC

[3] Joseph R. Blandino Ph.D., O.E., Benjamin H. Powell, Jr., '36, Institute professor, Department of Mechanical Engineering, Virginia Military Institute

[4] Blandino, J. R., Browne, H. M., Long, S. K., and Sterling, J., "A Primer on Photogrammetry for Ground Testing Scientific Balloons Using a ULDB Model", AIAA 2007-2633, *AIAA Balloon systems Conference*, Williamsburg, VA 2007.

[5] Blandino, J. R., Browne, H. M., Long, S. K., and Sterling, J., "Development of Ground Testing Methodologies for a 15 m ULDB Model", AIAA 2007-1849, *AIAA Balloon systems Conference*, Williamsburg, VA 2007.